- Area between curves (ch 6.1)

  (provided that on the interval $[a, b]$, $f(u) \geq g(u)$

  $$A = \int_a^b [f(u) - g(u)]\ du$$

  Essentially, $f(u) - g(u)$ needs to be {right function} − {left function} if the functions are in terms of y and you have dy.

  Otherwise, $f(u) - g(u)$ needs to be {upper function} − {lower function} if the functions are in terms of x and you have dx.

- Volume (ch 6.2, 6.3)
  - General:

    $$V(u) = \int_{u=a}^{u=b} A(u)\ du$$

  - Disk Method

    $$\text{rotation about x-axis: } V(x) = \int_{x=a}^{x=b} \pi\left(f(x)\right)^2\ dx$$
    $$\text{rotation about y-axis: } V(y) = \int_{y=a}^{y=b} \pi\left(g(y)\right)^2\ dy$$

  - Cylindrical shells

    $$\text{rotation about y-axis: } V(x) = \int_{x=a}^{x=b} (2\pi x)\left(f(x)\right)\ dx$$
    $$\text{rotation about x-axis: } V(y) = \int_{y=a}^{y=b} (2\pi y)\left(g(y)\right)\ dy$$

- Arc Length (ch 6.4)
  - If $x = f(t)$ and $y = g(t)$

    $$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

  - If $x = x$ and $y = g(x)$

    $$L = \int_{x=a}^{x=b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

  - If $x = f(y)$ and $y = y$

    $$L = \int_{y=a}^{y=b} \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\ dy$$

- Average Value (ch 6.5)

  $$f = \frac{1}{b-a}\int_a^b f(x)\ dx$$

- Work (ch 6.6)

  $$W = \int_a^b f_{orce}(x)\ dx$$

- Center of Mass (ch 6.6)
  - Moments

    $$M_y = \rho\int_a^b xf(x)\ dx$$
    $$M_x = \rho\int_a^b \frac{1}{2}\left[f(x)\right]^2\ dx$$

  - Centroid

    $$\bar{x} = M_y / \left(\int_a^b f(x)\ dx\right)$$
    $$= \frac{1}{A}\int_a^b xf(x)\ dx$$
    $$\bar{y} = M_x / \left(\int_a^b f(x)\ dx\right)$$
    $$= \frac{1}{A}\int_a^b \frac{1}{2}\left[f(x)\right]^2\ dx$$

- Economic Surplus (ch 6.7)
  - Consumer Surplus

    Given a "Production Level" of C, then $P = p(C)$ and Consumer Surplus is

    $$\int_0^C [p(x) - P]\ dx$$

  - Producer Surplus

    Given a "Production Level" of C, then $P = p(C)$ and Producer Surplus is

    $$\int_0^C [P - p(x)]\ dx$$

- Probability (ch 6.8)
  - Probability Density Function

    $f(x)$ is a probability density function if both of the following are true:

    * $f(x) \geq 0$ for all x
    * $1 = \int_{-\infty}^{\infty} f(x)\ dx$

  - Probability of an event

    If $f(x)$ is a probability density function, then

    $$\mathbb{P}\left(a \leq X \leq b\right) = \int_a^b f(x)\ dx$$

  - Average Value (Mean)

    if $f(x)$ is a probability density function, then the mean or average value is given by

    $$\mu = \int_{-\infty}^{\infty} xf(x)\ dx$$